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STUDY ON PROGRESSIVE COLLAPSE OF SINGLE-LAYER ARCH SHELL STRUCTURE BASED ON VUMAT STABILITY CONSTITUTIVE MODEL[1]

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In this paper, the constitutive model of material stability considering material damage evolution in Abaqus finite element software and the VUMAT stability constitutive model are used to simulate the progressive collapse progress of a single-layer latticed arch shell structure under an adverse load by applying the ultimate load. The results show that the progressive collapse of the single-layer latticed arch shell structure is mainly caused by material instability, and the damage range is large when the material stability constitutive model considering material damage evolution is adopted. When using VUMAT stability constitutive model, the progressive collapse form of the single-layer latticed arch shell structure shows a trend of extending from the top and bottom areas to the intermediate, and finally expanding from the intermediate to the vertical area of the single-layer latticed arch shell. Moreover, this kind of progressive collapse with a certain sequence due to destabilization damage of the bar is more in line with the actual engineering conditions, and it is convenient to describe the progressive collapse process in detail.

Keywords: single-layer latticed arch shell structure, progressive-collapse, VUMAT stability constitutive, damage evolution

1. Introduction

A latticed arch shell structure is a spatial framework which is based on bars and arranged according to the shell structure. It has properties of both bars and shells (Yin *et al.*, 1996). Due to the large span and sufficient space utilization of the single-layer latticed arch shell structure, the bars at both ends of the latticed arch shell easily lose stability due to the excessive axial force, which will affect bearing capacity of the whole latticed arch shell structure. Compared with a long-span multi-storey space steel structure, the single-layer latticed arch shell structure has low redundancy and is sensitive to defects (Sun, 2020; Li *et al.*, 2018; Pandey and Barai, 1997). Lattice shell structures are normally designed primarily for gravity loads as their lightweight nature is ideal for large spans (Cedrón and Elghazouli, 2021). Moreover, due to good symmetry of the latticed arch shell structure layout, once the bar is unstable, the bar at the symmetrical position will also be unstable, resulting in global instability of the structure. The bar instability and the global instability basically occur at the same time (Lv *et al.*, 2015; Tian *et al.*, 2014; Kani and McConnel, 1987; Su, 2006; Fan *et al.*, 2009). The latticed arch shell structure reflects

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the development of the times and the progress of technology, and it is the inheritance and development of culture, so that the latticed arch shell structure, such as the Sydney Opera House, modern stations and so on, has been of great importance (Jin *et al.*, 2015). Most collapse accidents of large-span spatial structures are caused by design flaws, improper construction or extreme weather. To avoid similar accidents, the progressive collapse resisting the capacity is gradually becoming an essential requirement in the design of spatial structures (Xu *et al.*, 2021).

In 1961, the Bucharest dome in Romania (single-layer spherical reticulated shell with span of 93.5 m and rise of 19.107 m) buckled due to local snow, the instability area continued to expand, and the structure finally overturned and collapsed as a whole. On January 18, 1978, a gymnasium with space grid structure in Hartford City, Connecticut, the United States buckled under the action of rain and snow load, then the damage expanded rapidly, and the whole structure crashed and landed instantly. According to the survey, the load at that time was only half of the design load. This event prompted the engineering community to pay attention to the progressive collapse instability of long-span spatial steel structures, breaking the one-sided understanding that spatial steel structures with high statically indeterminate degrees can prevent progressive collapse instability in the traditional concept (Wei, 2017). Progressive failure refers to continuous disintegration of a structure until partial or complete collapse, resulting in severe economic losses and mass casualties (Makkar *et al.*, 2022). So far, the progressive collapse research works are mostly focused on frame structures, while the progressive collapse research on long-span spatial steel structures, especially single-layer latticed arch shell structures, are relatively few. In order to consider the uncertainties involved in the structures (e.g., external load and material properties), which have great influence on the overall behavior of the structure under progressive collapse, many researchers conducted reliability and robustness assessments (Zhou *et al.*, 2022).

The progressive collapse of a single-layer latticed arch shell structure is a dynamic instability problem, and it is difficult to accurately describe its occurrence and development. Agarwal *et al.* (2001) focused on the vulnerability of structures, clarified the hierarchical relationship between components, then studied the vulnerability coefficient of the components and evaluated their importance. The VUMAT subroutine can specify a state variable to control the deletion of the unit. When the state variable is 1, the unit participates in the stress-strain calculation. When the state variable becomes 0, the unit is deleted from the model, so that the user can specify the instable criterion to simulate instability and fracture of materials (Tian *et al.*, 2014). Thus, the progressive collapse process of a single-layer latticed arch shell structure can be accurately described step by step by deleting the unstable bars. Starossek classifies the collapse into four types including a collapse caused by load redistribution, collapse caused by explosion or impact, collapse caused by instability and a collapse caused by multiple effects (Starossek, 2007).

In this paper, the ultimate load is obtained by a step-by-step loading method applied to a single-layer arch shell structure to simulate the dynamic response of a single-layer arch shell. In the calculation process, the VUMAT subroutine is used to define the specified instable criterion to simulate degradation of bar stiffness. An instable sequence of the single-layer latticed arch shell structure bars is used to accurately track the occurrence and development process of the progressive collapse. At the same time, the material stability constitutive model considering material damage evolution is used as a comparison group to study the progressive collapse process of the single-layer latticed arch shell structure. Here, the progressive collapse process of the single-layer latticed arch shell structure under these two different conditions is compared to verify the superiority of the VUMAT subroutine in the progressive collapse process, which is more in line with the actual project. And the self-programming VUMAT subroutine can automatically identify the instable bars in the buckling and post buckling states.

2. An analysis model of the single-layer latticed arch shell structure

2.1. Basic data

A single-layer latticed arch shell structure with skew area is used as the calculation model, with a span of 72.2 m and a length of 138 m. Q345 steel is mainly used with the elastic modulus of $2.06 \cdot 10^5$ MPa, Poisson's ratio ν of 0.3, density of $7860\,\mathrm{kg/m^3}$, yield strength of 310 MPa and a linear expansion coefficient of $12 \cdot 10^{-6}$°C. The bars of this single-layer latticed arch shell structure have box sections using beam elements, and the supporting columns at the bottom of the latticed arch shell bars are completely fixed. The overall model is shown in Fig. 1a, and the column position in the model is shown in Fig. 1b. The finite element used in this model is a one-dimensional beam element with the box section, which can bear a transverse force and bending moment.

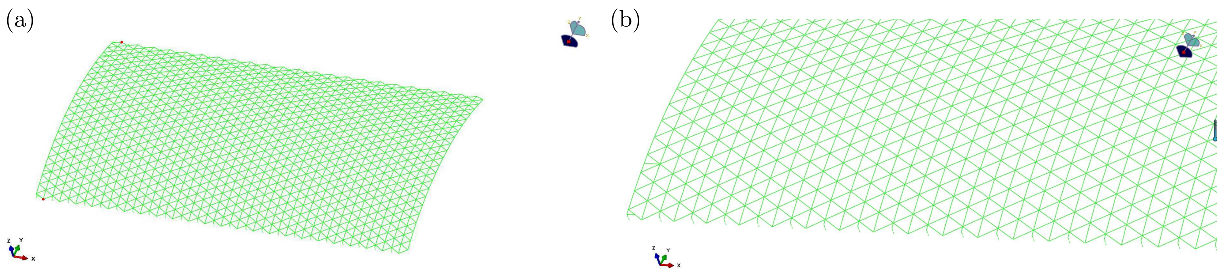

Fig. 1. (a) Latticed arch shell model, (b) Column location drawing

2.2. Material instability criteria

The material instability criterion is the strain value of the element exceeding the instability strain of steel ε_u to consider whether the unit fails or not. When the strain of the bar meets $\varepsilon \geqslant \varepsilon_u$, the element is judged to be invalid and automatically deleted from the calculation model in the process of finite element analysis. According to the requirements of current design specifications for the material stress and strain, when the material instability strain is $(1 \sim 1/15)\varepsilon_l$, where 1-1/15 is the range value, one to one fifteenth of the fracture strain ε_l, the maximum stress and maximum strain of the element in the structure change very little, which has little influence on numerical simulation of the progressive collapse. Therefore, the instability strain of steel is set $\varepsilon_u = 0.8\varepsilon_l = 0.02$. In this paper, the material stability constitutive model considering material damage evolution adopts this instability criterion. The damage evolution is shown in Fig. 2.

2.3. Bar instability criterion

The tension bar in the latticed arch shell structure reaches the yield strength and loses efficacy as strength breaks down. Compressed bars often fail because they can not reach the yield strength of the material due to instability, which is the loss of stability. Taking the compression bar stress exceeding the ultimate bearing capacity as the bar instability criterion, that is, when selecting the bar instability criterion, it is necessary to call the self-made VUMAT subroutine and define the transverse shear stiffness of the section. For the beam section, the calculation formula of the transverse shear stiffness is

$$K_{13} = K_{23} = kGA \tag{2.1}$$

where G is the shear modulus $G = E/2(1+\nu)$, A is the section area, k is the shear non-uniformity coefficient of the section. The value of K of common sections is shown in Table 1.

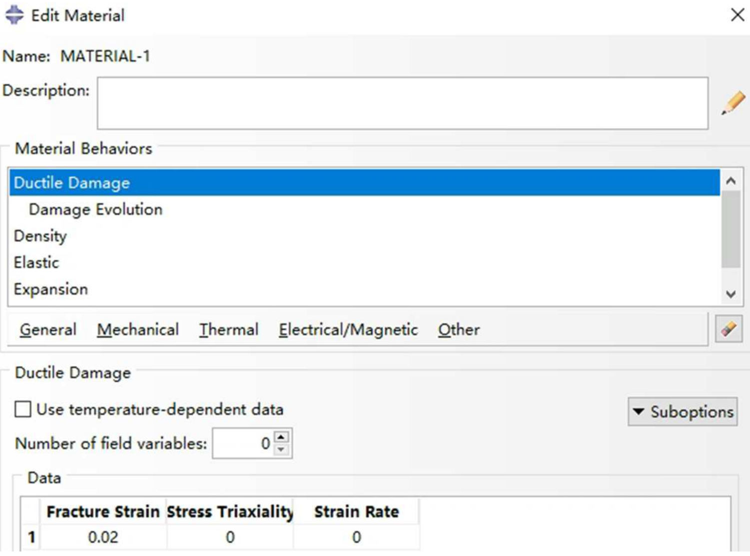

Fig. 2. Damage evolution diagram

Table 1. Shear non-uniformity coefficient of section

Section type	Shear non-uniformity coefficient K
Box type	0.44
Circular	0.89
Type I/T	0.44
Ring type	0.53
Rectangle	0.85

In this paper, the VUMAT stability constitutive model adopts this instability criterion, and the non-linear deformation and influence of the axial force on bending are considered.

3. Calculation results and analysis of the progressive collapse process

3.1. Material stability constitutive model considering material damage evolution

3.1.1. Node displacement time history curve

The displacement curve of node 87 in the area most prone to bar damage at the right end of the latticed arch shell is selected to visually reflect the progressive collapse process of the reticulated shell structure caused by bar instability under 10 times of self-weight, as shown in Fig. 4. And the loading time is shown in Fig. 3.

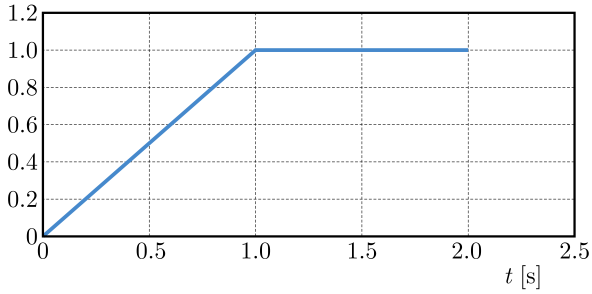

Fig. 3. Load-time diagram

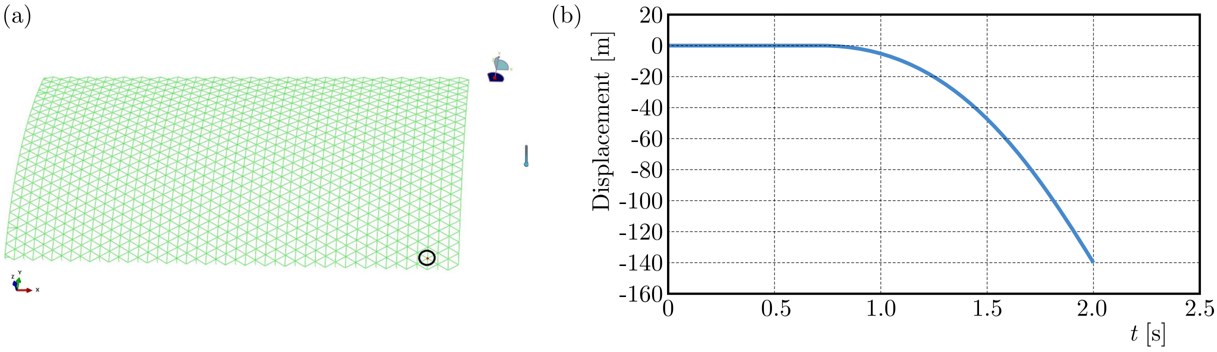

Fig. 4. Displacement curve

It can be seen from Fig. 4 that the displacement of the node in the edge area of the latticed arch shell structure changes greatly due to the instability of the top bar. At 0.9 s, the displacement of the node has begun to change. During the period from 1 s to 2 s, the latticed arch shell collapses progressively due to successive instability of the intermediate bar of the latticed arch shell, resulting in a rapid downward collapse of the node. The node displacement reaches the maximum value of 145 m in this 1 s period, indicating that the latticed arch shell structure has been completely destroyed at this time. Therefore, the node displacement changes slowly during 1 s-1.2 s, mainly in the initial stage of bar instability. During the period of 1.3 s-1.6 s, the change of node displacement accelerates, mainly in the stage of progressive bar instability. During the period of 1.7 s-2 s, the change speed of node displacement becomes the fastest, which is the stage of structural collapse. Node 87 is selected mainly because it is located in the right edge of the structure, which can accurately describe the displacement change of the structure when the collapse occurs.

3.1.2. Initial bar instability

The period 1 s-1.2 s is the initial bar instability stage, and the global deformation is shown in Fig. 5-Fig. 9.

It can be seen from Fig. 5 that the bar at the right end of the latticed arch shell structure has collapsed downward at 1 s, and the maximum value reached 6.04 m, but there was no bar damage at this time. In the middle part of the latticed arch shell structure, some bars have reached the maximum stress and began to yield.

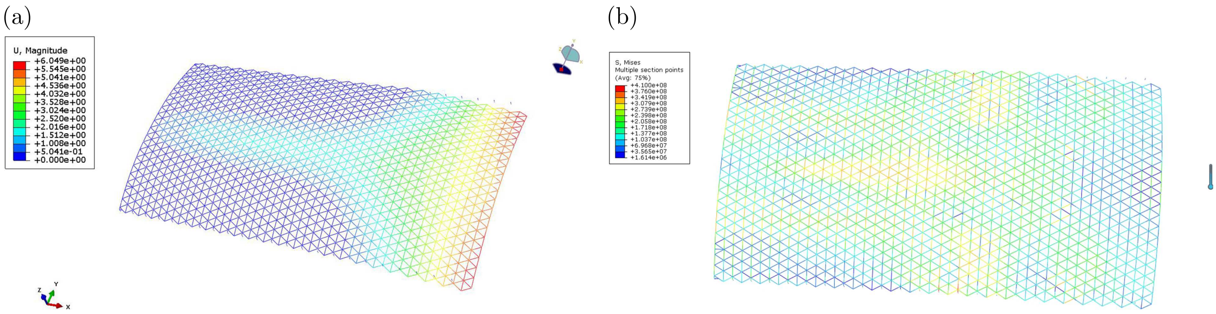

Fig. 5. Global deformation diagram at 1 s: (a) Z-direction displacement diagram, (b) stress diagram

At 1.1 s, the intermediate part of the latticed arch shell structure has some bars already started to fail and had a tendency to expand to the intermediate part, and the maximum displacement of the right end part of the latticed arch shell structure reached 11.6 m at this time.

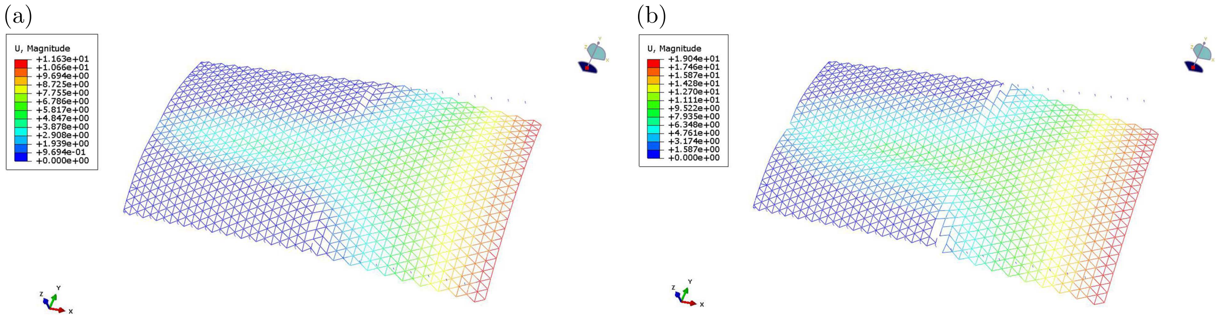

Fig. 6. Global deformation diagram at: (a) 1.1 s, (b) 1.2 s

It can be seen from Fig. 6b that at 1.2 s, a wide range of bar instability has occurred in the latticed arch shell structure, and it started from the intermediate part and approached the intermediate from the top and bottom ends. As the instability range became larger, the left end structure began to show a small range of the bar instability, while the maximum displacement of the right end bar progressive collapse reached 19m at this moment.

3.1.3. Progressive bar instability

Between 1.3 s-1.6 s the progressive bar instability stage occured, and the global deformation is shown in Fig. 7.

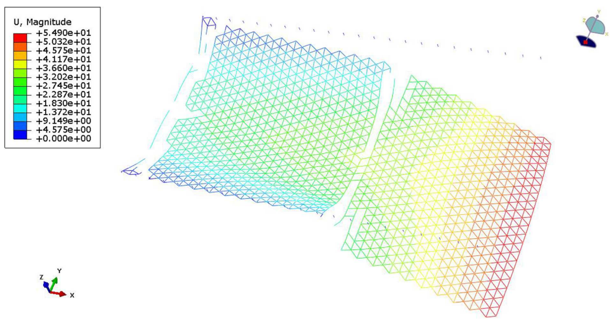
Fig. 7. Global deformation diagram at 1.5 s

It can be seen from Fig. 7 that during the period of 1.3 s to 1.6 s, the instability and damage of the bars are further expanded. The bars in the intermediate part are basically completely damaged, and the bars at the left end are also damaged in a large range with the passage of time.

3.1.4. Structural collapse

Within 1.7 s-2 s the structural collapse stage appeared, and the global deformation is shown in Fig. 8.

It can be seen from Fig. 8 that after 1.6 s, the intermediate part of the single-layer latticed arch shell is completely broken with the bar at the right end due to progressive bar instability. The latticed arch shell structure is divided into two parts: the left and right end, and the maximum displacement of the right end reaches 152 m (as shown in Fig. 5). It can be seen that considering the constitutive model of material stability and damage evolution, the progressive collapse of the bars is relatively rapid and the instability range is large. The left end of the latticed arch shell also protrudes upward due to the progressive downward collapse of the right part.

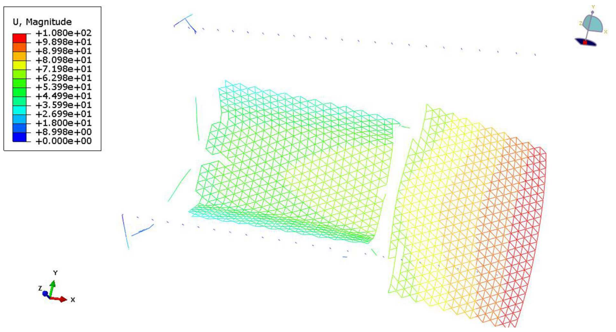

Fig. 8. Global deformation diagram at 1.8 s

As was stated above, in the material stability constitutive model considering material damage evolution under ultimate loads, the latticed arch shell bars begin to lose stability due to material instability in the intermediate part. The instability expands from the top and bottom sides to the intermediate one and induces bar material instability at the left edge without a certain law.

3.2. VUMAT stability constitutive model

3.2.1. Node displacement time history curve

Select now the node displacement curve of the bar most prone to instability in the edge area of the latticed arch shell structure to intuitively reflect the progressive collapse process of the structure caused by the bar instability under the action of the lead equal to 10 times of self-weight, as shown in Fig. 10.

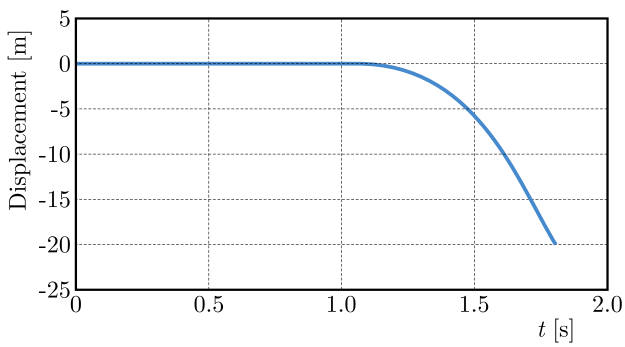

Fig. 9. Displacement curve

It can be seen from Fig. 9 that the displacement of the node at the edge of the latticed arch shell structure changes greatly due to the instability of the top bar. At 1.1 s, the displacement of the node has begun to change. From 1.2 s to 1.8 s, as the latticed arch shell edge bars successively reach the maximum yield strength and become unstable, the VUMAT subroutine removes the unstable bars and the latticed arch shell collapses progressively, causing this node to collapse rapidly downward. As a result, the node collapses rapidly downward, and the node displacement reaches the maximum value of 20 m in this 0.6 s time, indicating that the latticed arch shell structure has been completely damaged at this time. Therefore, the node displacement of the latticed arch shell structure changes slowly during the period of 1.1 s-1.2 s, mainly in the initial bar instability stage. During the period of 1.3 s-1.5 s, the node displacement changes are accelerated, mainly in the stage of progressive bar instability. During 1.6 s-1.8 s, the node displacement changes fastest, which is the stage of structural collapse.

3.2.2. Initial bar instability

The progressive collapse process of the single-layer latticed arch shell structure at 1.1 s and 1.2 s is shown in Fig. 10 and Fig. 11, respectively.

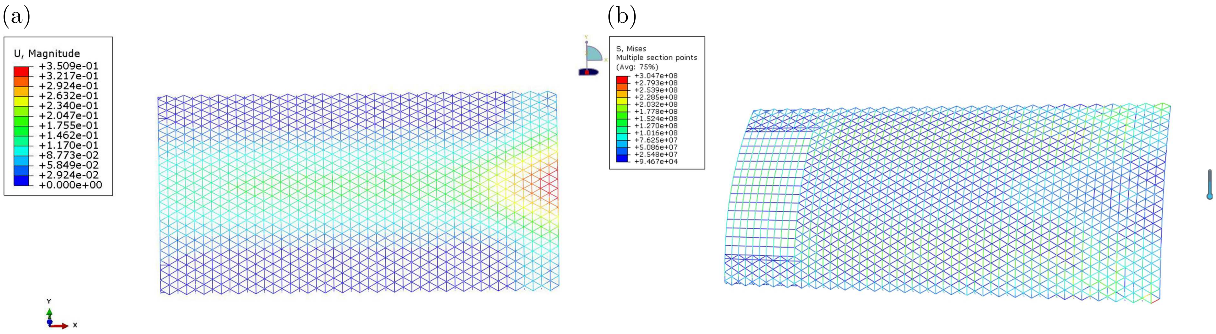

Fig. 10. Global deformation diagram at 1.1 s: (a) Z-direction displacement diagram, (b) stress diagram

It can be seen from Fig. 10 that at 1.1 s, the displacement of the bar at the right edge has changed by 35 cm, but there was no instability of the latticed arch shell bar at this time. At this moment, the bars did not reach the yield state.

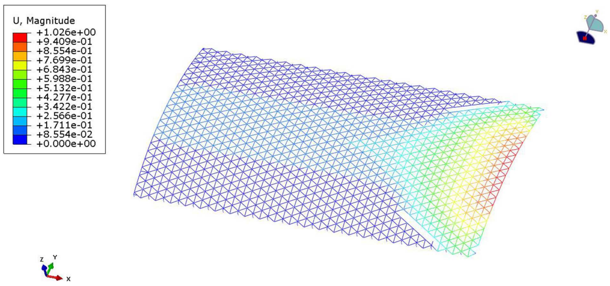

Fig. 11. Global deformation diagram at 1.2 s

It can be seen from Fig. 11 that at 1.2 s, the latticed arch shell structure has experienced bar instability. The top bar in the state of instability was judged to be failed and removed, whereas the right end of the top and bottom sides of the bar showed a trend of damage to the intermediate part. At this time, the maximum displacement of the right end bar has reached 1.02 m. Comparison with the material stability constitutive model considering material damage evolution reveals that this model presents a progressive collapse due to the instability of latticed arch shell bars. The degree of bar collapse is small, while the previous constitutive model described a progressive collapse caused by material instability of intermediate bars, and the degree of bar collapse is large.

3.2.3. Progressive bar instability

The progressive collapse process of the single-layer latticed arch shell structure at 1.3 s and 1.5 s is shown in Fig. 12a and Fig. 12b.

By comparison, it is found that before 1.5 s, the bar instability of the latticed arch shell structure shows an expanding pattern from the top and bottom sides to the intermediate part, and gradually extending to the left end, indicating that the latticed arch shell structure at this time still has a certain bearing capacity. At this stage, due to the large range of material instability, the left end bar has been deleted due to this instability and then collapsed downward.

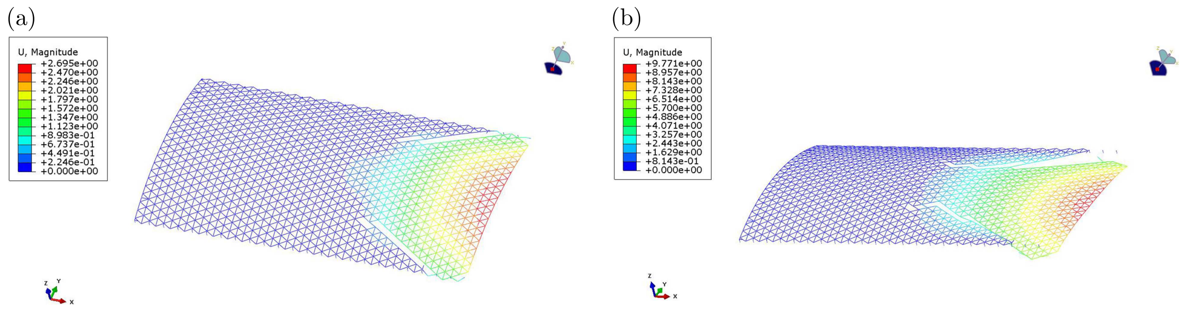

Fig. 12. Global deformation diagram at: (a) 1.3 s, (b) 1.5 s

3.2.4. Structural collapse

The progressive collapse process of the single-layer latticed arch shell structure after 1.5 s is shown in Fig. 13a and Fig. 13b.

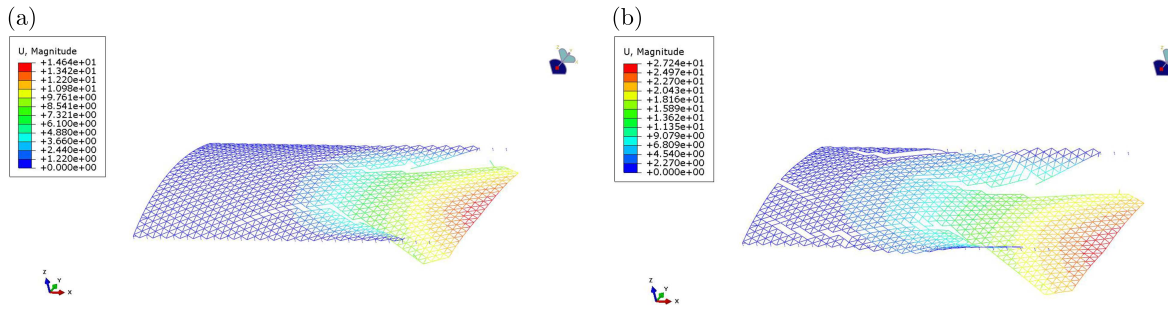

Fig. 13. Global deformation diagram at: (a) 1.6 s, (b) 1.8 s

It can be seen in Fig. 13a and Fig. 13b that after 1.5 s, the instability of latticed arch shell bars shows irregularity, and the instable range is all over the whole latticed arch shell structure. At this time, the latticed arch shell structure is also unstable due to too many bars, and the collapse displacement reaches 27 m, indicating that the latticed arch shell structure has been completely damaged. The previous material constitutive model has completely collapsed and destroyed.

As was stated above, according to the VUMAT stability constitutive model, the compression bars of the latticed arch shell are damaged first. And the bar at the beginning of the damage showed a regular form of damage from the top and bottom sides of the right end to the intermediate part, while after 1.5 s, the damage showed a large range and irregular form due to the complete loss of load-bearing capacity of the latticed arch shell. And the top and bottom bars protrude upward due to downward collapse of the intermediate bar.

4. Conclusions

Based on the analysis of the progressive collapse process of a single-layer latticed arch shell structure under the ultimate load by using the VUMAT stability constitutive model and the material stability constitutive model considering material damage evolution, the following conclusions are drawn:

- When using the VUMAT stability constitutive model, it can be found that the instability sequence of bars of the latticed arch shell structure shows an extending trend from the top and bottom sides to the intermediate part, and finally from the intermediate part to the left end of the single-layer latticed arch shell, which has a strong regularity, indicating that this constitutive model presents the progressive collapse caused by the bar instability. For the latticed arch shell structure with the material stability constitutive model considering material damage evolution, the extent of bar destabilization damage occurs initially only in the intermediate region of the bar, and the damage is also more severe, and is mainly due to the progressive collapse of the material as it reaches its yield strength.

- The bar instability of the latticed arch shell structure occurs for the first time in the VUMAT stability constitutive model at 1.2 s, and the reinforcement model considering damage evolution occurs at 1.1 s. In the initial bar instability stage, the collapse degree of the VUMAT stability constitutive model is not very large. Due to gradual instability of other bars, the collapse degree expands slowly. The material stability constitutive model considering material damage evolution shows that the collapse degree is already very large in the initial bar instability stage. It can be seen that the VUMAT stability constitutive model can describe the progressive collapse process caused by the latticed arch shell bars in more detail, and is closer to the structural dynamic response of the latticed arch shell structure in engineering applications. Therefore, in practical engineering applications, it is necessary to strengthen the initial stablity of the bars, by making use of e.g. of higher strength steel.

The VUMAT subroutine is optimized to be more suitable for different materials in the future.

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